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Numerical Study for the Temperature Distribution in an Incompressible Fluid through Porous Medium

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ABSTRACT
In this paper we analyses the transient, radial temperature distribution in a porous medium for a constant-rate injection of an incompressible fluid from a wellbore. The energy transfer is included the formation by conduction and convection and appropriate boundary conditions of Danckwerts are applied at the finite- radius wellbore. By using similarity transformations, the governing equation can be converted in ordinary differential equation and it can be solved numerically by B-Spline Collocation technique as well as Galerkin technique.

KEYWORDS: B-Spline Collocation technique, Galerkin technique, Radial temperature, Transient temperature, porous medium

I. INTRODUCTION

The Heat and mass transport problems consist a transient convection-diffusion equation. There are many applications like thermal methods for oil recovery, washing of filter cakes, thermal energy storage in packed beds and in permeable formations, fluid and thermal analyses of geothermal reinjection, miscible flooding and energy extraction from hot rock. An analytical solution to describe longitudinal mixing of solvent and solute phases in a porous medium was described by Brenner [1]. An analytical solution with the addition of longitudinal diffusion was obtained by Avdonin [2]. The problem of hot fluid injection in an oil reservoir was analyzed by Lauwerier [3] in the absence of longitudinal thermal diffusion. An analytical solution for the response of a packed bed thermal storage system to a step change in inlet temperature was derived by Riaz [4]. A fundamental problem was pointed out by Laumback [5] which arises when the convection diffusion equation is solved numerically.

An increased attention has been received by combined convection diffusion problems as applied to geothermal reservoirs. Numerical solutions are obtained which can be treated the general three dimensional problem. At the time of these reservoirs simulation codes, it is necessary to have known solutions for code verification. The numerical solutions were compared by Faust and Mercer [6] to the one-dimensional analytical results of Avdonin [2]. There are many examples of convection-diffusion problems of Oil recovery by hot fluid injection, geothermal production and reinjection, miscible flooding, energy storage in permeable formations and energy extraction from hot rock using huff and puff injection extraction techniques. These problems all involve transient, radial convection and diffusion. Normally, the solution is self-similar with simple form. The early-time convergence to the similarity solution is investigated numerically.

- **FORMULATION**

There is a horizontal porous layer whose thickness is h and it is bounded above and below by relatively impermeable layers as shown in figure 1. A vertical wellbore of radius intersects the porous layer. Fluid at temperature is injected into the formation at a constant volume flow rate, Q. Let the given fluid is incompressible with constant heat capacity and to progress uniformly into the porous medium displacing the previous pore fluid. In local thermal equilibrium, the fluid and porous solid are assumed, which expected in geological applications where the characteristic micro scale dimension is small and fluid velocity is not too much. Heat conduction losses to the bounding impermeable layers are neglected. As is normally in porous media calculations, the hydrodynamic dispersion of the velocity and temperature fields due to non-homogeneity of the medium are neglected.

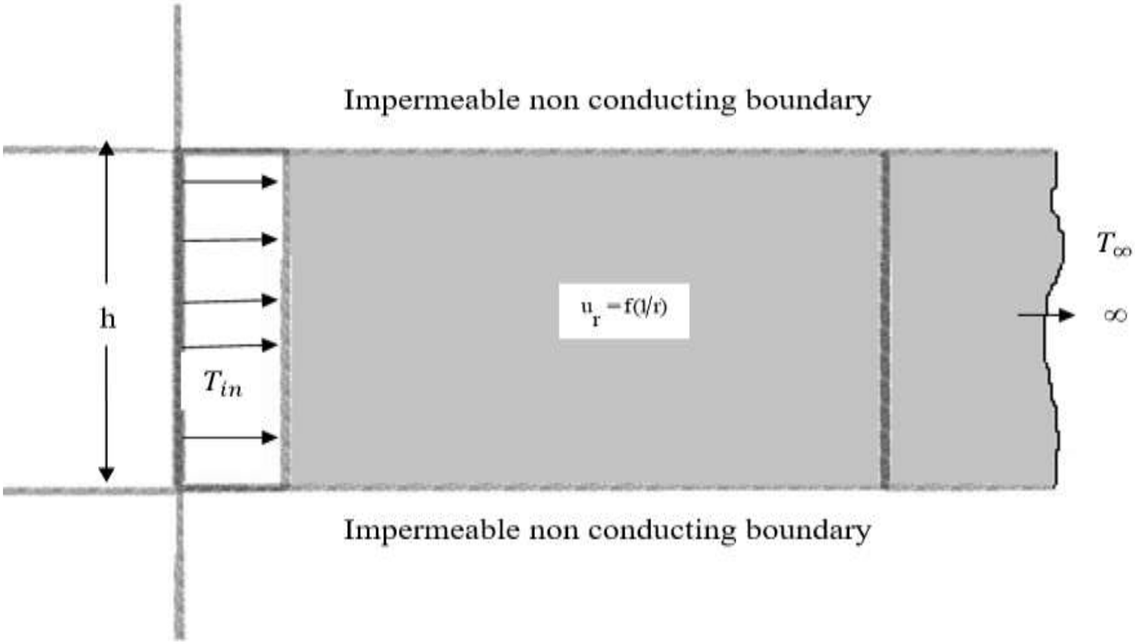

Fig 1 : Redial well injection problem

As described above, the temperature and the velocity in the porous layer different in the radial direction. Therefore the energy equation is expressed as,
(1)
where = Fluid heat capacity, = Thermal conductivity and= Thermal diffusivity.
These effectual thermal properties are normally defined by,
where is a porosity.
In the beginning, the temperature is uniformly distributed . When time T = 0, we start injecting fluid having given temperature inside the wellbore. The fluid temperature, T(0,t), by the side of the wellbore surface progressively rises in the direction of T_{in}, in compliance
with Danckwert's boundary condition,at
which shows that the energy balance is satisfied at the porous surface. Since we consider semi-infinite medium in the radial direction, as showing that the temperature approaches its uninterrupted value. The non- dimensional form can express by the energy equation in the form of (2)

where the boundary conditions
at as
(3)
Here, the dimensionless parameters are described as follows:

where is the Peclet number. It characterizes the proportion between convective and conductive transport of energy. Equation (2) and (3) are linear in . The related equation is obtained for the constant density miscible displacement of one fluid by another. We replaced the temperature in equation (2) by concentration of the injected fluid, the Reynolds number product replaces P_e , becomes the mass diffusion coefficient D and becomes . Here became very small. So, we can show that . Using similarity solution, Partial differential equation (2) reduces to ordinary differential equation as

(4)
With boundary condition:

(5)

- **SOLUTION BY B-SPLINE COLLOCATION METHOD**

Let is an approximate solution of (4). Where the unknowns ; j = -1,0,1,2,......, n+1 to be determined and is the cubic B-Spline [8] at different knots of the partition of [0 2]. We obtain the equations:

(6)
(7)

(8)

Now for n = 4 (i.e. h = 0.5) the knots of our partition are . Evaluating and its derivatives at these knots by virtue of B-Spline values and equations (6), (7) and (8) we have obtain a system of seven linear equations in seven unknowns as follows:

(9)

(10)

(11)

Now, from equation (9), (10) and (11) we obtain the system of seven equation in seven unknowns of the form

A X = B

(12)

Solving this system of equations, we obtain the values of unknowns Substituting all these values in s (t) we obtain the complete solution of the problem as shown in figure 1 for different values of .

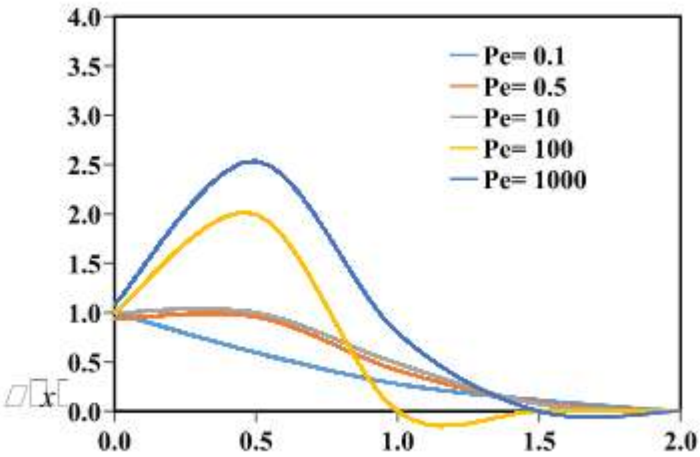

Fig 2 : Radial temperature distributions by B-Spline Collocation Method

- **SOLUTION BY GALERKIN METHOD**

We take as before to satisfy the boundary conditions (Four term solution).

(13)

So that,

The residue is,

We now carry out four integrations for four unknowns C_1, C_2, C_3 and C_4.

Using as a :

(14)

Using as a :

(15)

Using as a :

(16)

Using as a :

(17)

Then we derived four equations in terms of C_1, C_2, C_3 and C_4 for

= 0.1

$(-27.347607)C_1+(-37.320663)C_2+C_3(-65.121201)+C_4(-110.773598)=14.86381$

(18)

$(-44.119835)C_1+(-58.473518)C_2+C_3(-85.689522)+C_4(-163.630264)=23.740002$

(19)

$(-74.080795)C_1+(-96.502495)C_2+C_3(-138.007217)+C_4(-260.596588)=39.659794$

(20)

$(-127.806053)C_1+(-164.569794)C_2+C_3(-231.698395)+C_4(-433.789185)=68.213005$

(21)

A X = B

Hence the approximate solution is given by,

The radial temperature distribution during fluid injection is presented in figure 3 for different values of through Galerkin technique.

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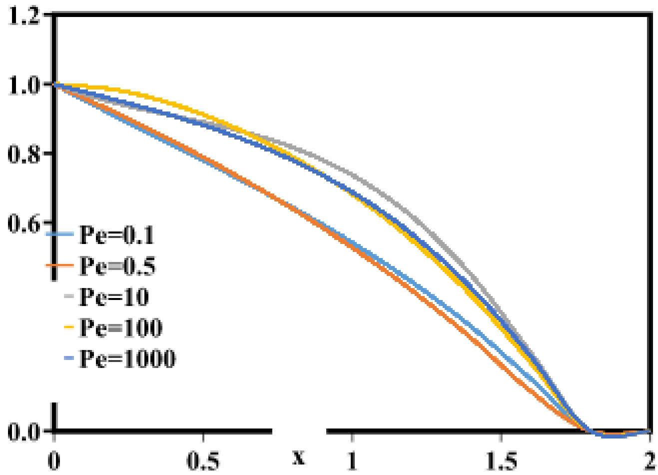

Fig 3 : Radial temperature distributions by B-Spline Collocation Method

Graphical results for P_e = 0.1, 0.5 and 10 are shown in Figures 4,5,6,7,8 and 9 by Cubic B-Spline Collocation and Galerkin method[10],[11] respectively. For all x, when , at the injection temperature is suddenly increased to , and the fluid temperature in the porous medium begins to rise. For very small times, the radial divergence of the coordinate system is insignificant, and the solutions are in good agreement with the one dimensional Cartesian results of Riaz [4]. The early behavior should be diffusion dominated and, hence, the penetration depth, , grows like for sufficiently small t. When stated in terms of the similarity variable, the early time penetration depth is roughly

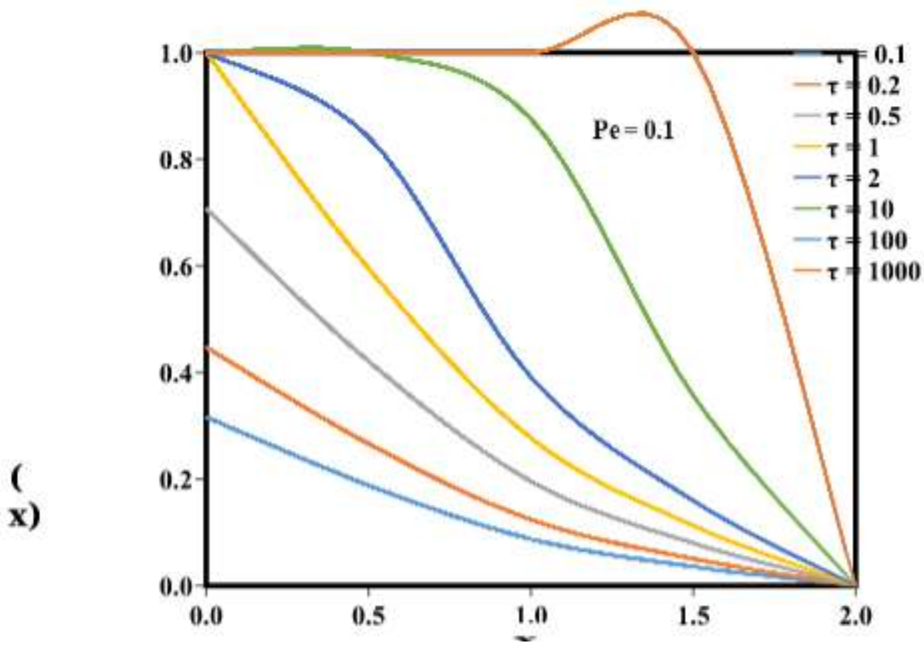

Fig 4 : Transient temperature distributions by B-Spline Collocation Method Method for Pe = 0.1

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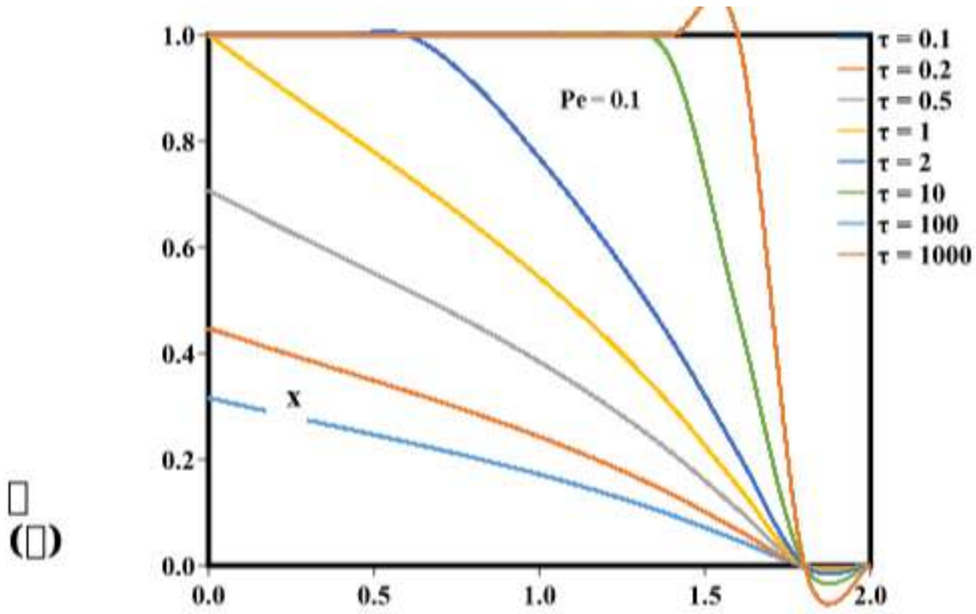

Fig 5 : Transient temperature distributions by B-Spline Collocation Method Method for Pe = 0.1

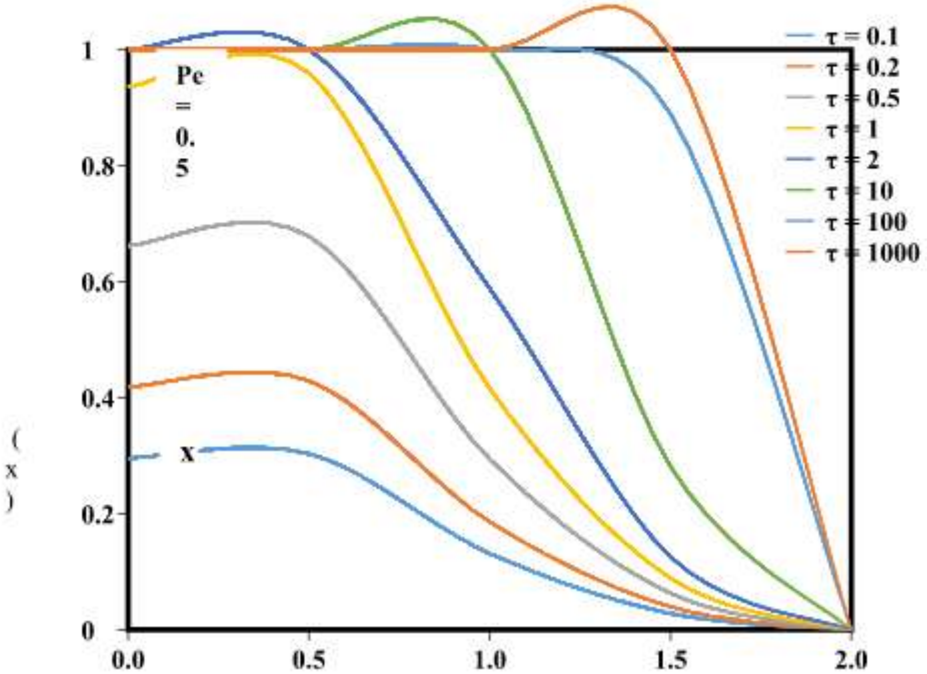

Fig : 6 Transient temperature distributions by Cubic B-Spline Collocation Method for Pe = 0.5

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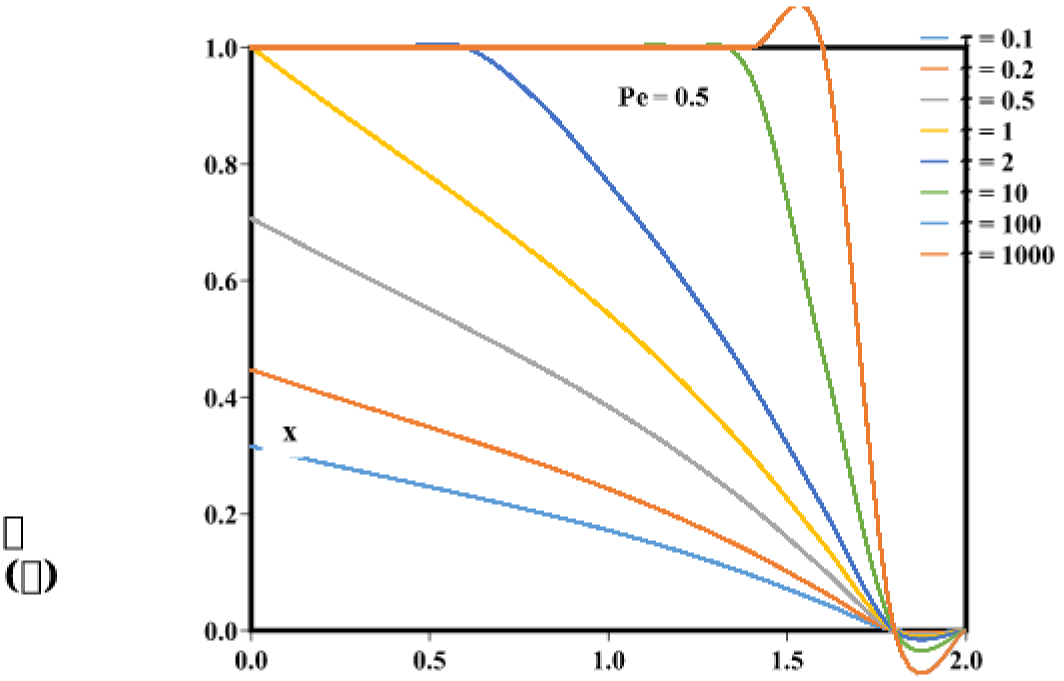

Fig 7: Transient temperature distributions by Galerkin Method for Pe = 0.5

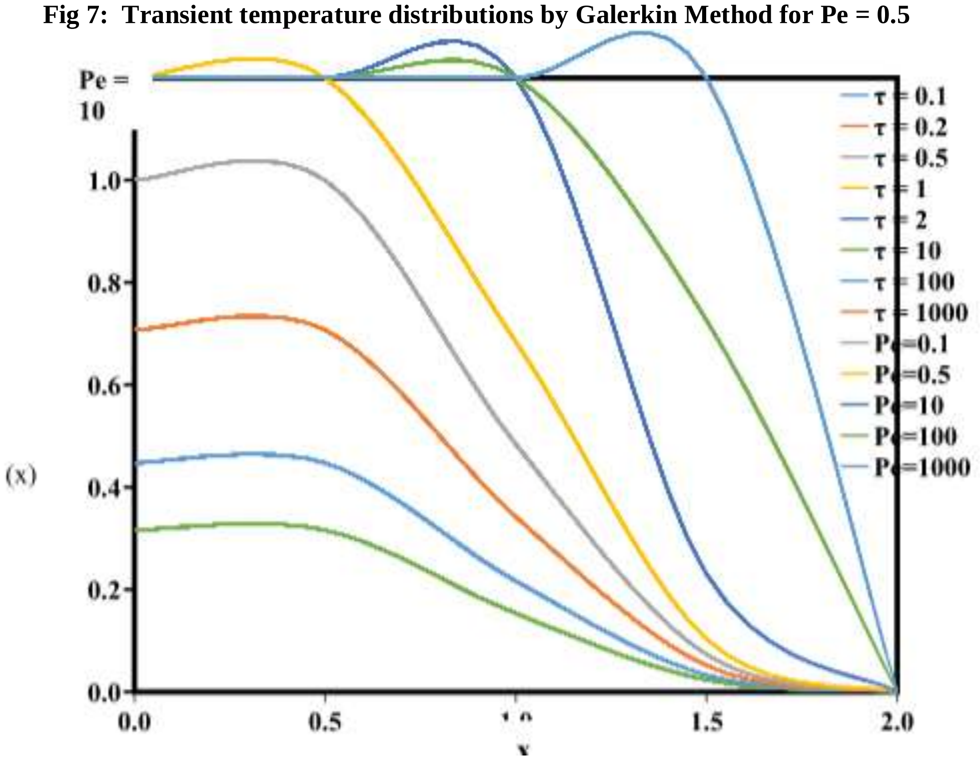

Fig 8 : Transient temperature distributions by Cubic B-Spline Collocation Method for Pe = 10

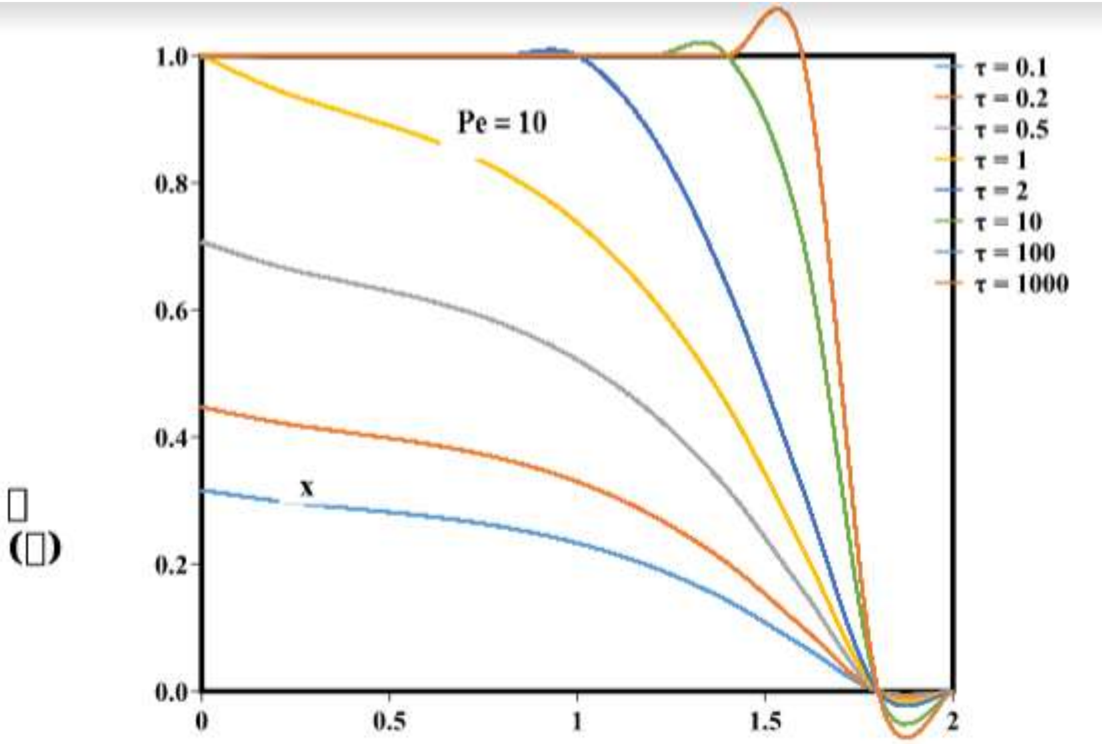

Fig 9 : Transient temperature distributions by Galerkin Method for Pe = 10

By comparing the Figures 4,5,8 and 9, it is observed that the solutions become identical as , which is the range of greatest practical integer. The transient convection-diffusion equation is solved. In one-dimensional radial coordinates, the transient convectiodiffusion equation can be solved. Graphical results are found for Pe = 0.1,0.5 and 10 by Galerkin method and Cubic B-Spline Collocation method over a wide range of dimensionless time. These solutions give the early-time behavior to similarity solutions which are valid for most times of interest.

II. CONCLUSION

For reasonable fluid injection rates where Peclet number is usually well above unity, the similarity solution describes the transient, radial temperature field for most times of interest. At very late times and in shallow layers, the penetration depth of the thermal front will ultimately become controlled by conduction losses to the impermeable strata which lie above and below. Thus the numerical solution of the problem provides satisfactory results that are in agreement with the natural phenomena which emphasizes the reliability of the said numerical technique.

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